9 786029 534306

Department of Mathematics
Faculty of Mathematics and Natural Sciences,
Andalas University, Indonesia

ICMSA 2009

June 9-11, 2009 The Hills Hotel Bukittinggi, Indonesia

PROCEEDING

The 5th IMT - GT International Conference on Mathematics, Statistics and Their Applications

ICMSA 2009

Editors :
I Made Arnawa, Muhafzan, Maiyastri, Susila Bahri

June 9-11, 2009
The Hills Hotel
Bukittinggi, Indonesia

Organized by :

Proceedings of

The 5^{th} IMT-GT

International Conference on Mathematics, Statistics, and their Applications 2009

(ICMSA 2009)

"Mathematics and Statistics for Industry and Community Development"

June 09 – 11, 2009

The Hills Hotel, Bukittinggi - Indonesia

Organized by :
Department of Mathematics, Andalas University,
Padang – Indonesia

Published by :

Department of Mathematics, Andalas University
Kampus UNAND Limau Manis
Padang 25163, Sumatera Barat,
INDONESIA.

ISBN 978-602-95343-0-6

June 2009

Preface

First of all, I would like to say welcome to Bukittinggi, Indonesia to all of you. It is an honour for us to host this conference. We are very happy and proud because the participants of this conference come from many countries; we have participants from Libya, Japan, Qatar, India, Malaysia, Singapore, Thailand, Iran, and many more.

Ladies and gentlemen, according to constructivism theory, mathematics comes out as a result of social construction; that's why, the outcomes of our researches in mathematics, like theorem or formula of mathematics, should be communicated in a scientific forum such as seminar or conference. Through this kind of seminar or conference, we could refine the existing theorems or we could get new ideas to produce a new one. Seminar or conference which is held annually enables us to continually develop the science of mathematics until the end of the time.

That's way, in this two-day conference, we are going to discuss around 250 papers coming from diverse aspects of mathematics ranging from analysis, applied mathematics, statistics, algebra, Computational Mathematics, mathematics education, and other related topics.

For all of us here, I would like to convey my endless appreciation and gratitude for your participation in this conference.

Thank you very much

Dr. I Made Arnawa
Chairman of the Conference

Message from Rector Andalas University

It gives me great pleasure to extend my sincere and warm welcome to the participants of the 5th International Conference on Mathematics Statistics and Application (The IMT GT's 5th ICMSA 2009) - A Joint Scientific Program organized by Universities over Indonesia, Malaysia and Thailand Growth Triangle Region. On behalf of Andalas University, let me welcome all of you to the conference in Bukittinggi, West Sumatra Province, the land of Minang kabau.

We believe that from this scientific meeting, all of participants will have time to discuss and exchange ideas, findings, creating new networking as well as strengthen the existing collaboration in the respective fields of expertise. In the century in which the information is spreading in a tremendous speed and globalization is a trend, Andalas University must prepare for the tough competition that lay a head. One way to succeed is by initiating and developing collaborative work with many partners from all over the world. Through the collaboration in this conference we can improve the quality of our researches as well as teaching and learning process in mathematics and to achieve standards and requirements applied in many developed countries. I strongly believe that this conference is and extraordinary testimony to our capacity building at international, regional and local level.

I would like to express my deep gratitude to International Scientific Committee of who has honored the Mathematics Department, Faculty of Mathematics and Natural Sciences, Andalas University to host this prestigious conference. This is a very special opportunity for us to be involved in a regional community of knowledgeable scientist in the field of mathematics, statistics and their applications. I would also like to extend my gratitude to keynote speakers, participants, and organizer of this conference for their contribution to this event. My special thank is also rendered to the local government of West Sumatra for various supports and facilities.

Finally I wish all participants a fruitful deliberation at the conference. I also wish all participants and accompanying spouses a pleasant and enjoyable stay in Bukittinggi City, West Sumatra.

Prof. Dr. Ir. Musliar Kasim, MS
Rector

ORGANIZING COMMITTEE

Patrons : Rector Andalas University
Vice Rector of Academic Affair Andalas University

Steering : Dean Faculty of Sciences Andalas university
Vice Dean of Academic Affairs Andalas University

International Scientific Committee

Prof. Dr. Novesar Jamarun (Andalas University – Indonesia)
Prof. Dr. Herman Mawengkang (University of Sumatera Utara – Indonesia)
Prof. Dr. Dato Rosihan Ali (University Sains Malaysia)
Assoc. Prof. Dr. Ahmad Izani Md. Ismail (University Sains Malaysia)
Assoc. Prof. Dr. Putipong Bookkamana (Chiang Mai university – Thailand)
Assoc. Prof. Dr. Anton Abdulbasah Kamil (University Sains Malaysia)
Assoc. Prof. Dr. Pachitjanut Siripanitch (School of Applied Statistics – Thailand)
Dr. Saib Suwilo (University of Sumatera Utara – Indonesia)
Dr. Hizir Sofyan (Syiah Kuala University – Indonesia)
Dr. Tarmizi Usman (Syiah Kuala University – Indonesia)

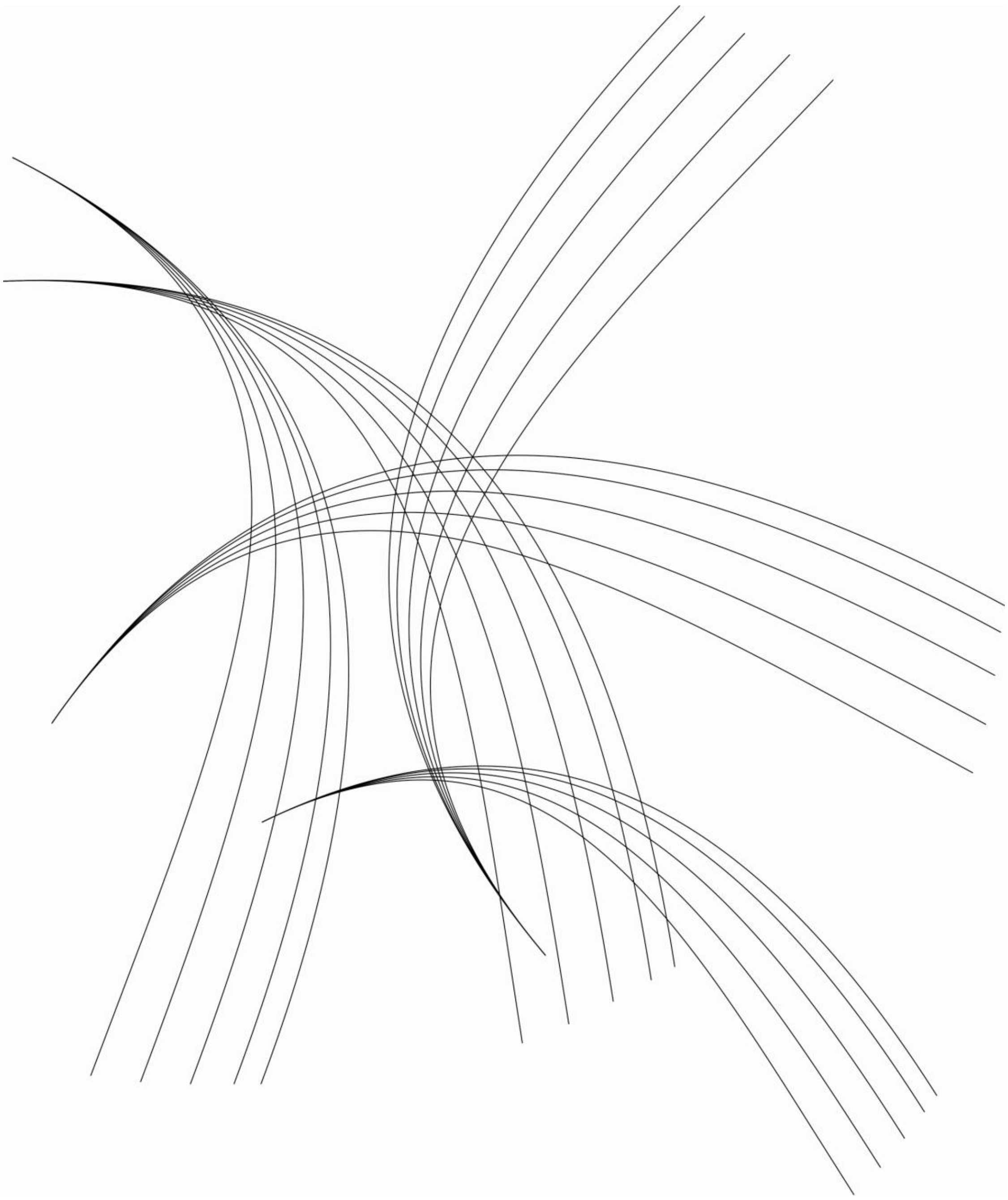
Local Scientific Committee

Dr. Maiyastri (Andalas University – Indonesia)
Dr. Susila Bahri (Andalas University – Indonesia)
Dr. Muhafzan (Andalas University – Indonesia)
Adek Tasri, Ph.D (Andalas University – Indonesia)

Committee

Chairman	:	Dr. I Made Arnawa.
Secretary	:	Ir. Yudiantri Asdi, M.Sc.
	:	Izzati Rahmi, H. G., M.Si.
Transportation/Accomodation	:	Drs. Syafruddin, Zulakmal, M.Si.
Treasurer	:	Ir. Werman Kasoep, M.Kom., Jenizon, Msi, Ir. Hazmira Yozza, M.Si., Ir. Insanul Kamil, MS., Drs. Zulkarnain Khaidir, MS., Drs. Mulyadi, MS.
Technical Programme	:	Dr. Muhafzan, Dr. Maiyastri, M.Si.
Secretariat / Documentation	:	Budi Rudianto, M.Si., Narwen,M.Si., Arrival Rince Putri, M.T., Budi Rahmadya, S.Kom, Mohammad Hafiz Hersyah, S.Kom., Rahmi Eka Putri, S.Kom., Ratna Aisuwarya, S.Kom, Fajar Wisga, Berkah Fajar.

Contents



CONTENTS

Applied Mathematics **445 - 979**

Applications of Fuzzy Number Max-Plus Eigenvalues on Queuing Networks with Fuzzy Activity Times

M. Andy Rudhito[1*], Sri Wahyuni[2], Ari Suparwanto[3], and F. Susilo[4]

[1]Ph.D student in Department of Mathematics, Gadjah Mada University
Sekip Utara Yogyakarta
[2,3]Department of Mathematics, Gadjah Mada University
Sekip Utara Yogyakarta
[4]Department of Mathematics , Sanata Dharma University
Paingan Maguwoharjo Yogyakarta
*e-mail : [1]rudhito@staff.usd.ac.id

Abstract

The activity times in a network is seldom precisely known, and then could be represented into the fuzzy numbers. This paper aims to determine the service cycle completion time of the acyclic fork-join queuing networks with fuzzy number activity times using fuzzy number max-plus algebra. This paper is a theoretical investigation based on literature and computation using MATLAB program. The finding shows that the service cycle completion time is a eigenvalue of matrices over fuzzy number max-plus algebra in the system.

Keywords: Max-Plus Algebra, Queuing Networks, Fuzzy Number, Completion Times, Eigenvalues.

1. Introduction

The max-plus algebra can be used to model and analyze a network, like the project scheduling, production system, queueing networks, etc [1], [2] and [3]. The networks modeling with max-plus algebra approach is usually a max-plus linear system equations and it can be written as a matrix equation. The periodical properties of networks dynamics can be analyzed through the max-plus eigenvalues and eigenvectors of matrices in its modelling. In [3] and [4] have been discussed an algebra model of the acyclic fork-join queuing networks with real (crisp) activity times using max-plus algebra into a system of max-plus linear equations. The service cycle completion time of the networks is an eigenvalue of matrix in the system have been discussed in [4]. In [5] have been discussed an algebra model of the acyclic fork-join queuing networks with interval activity times using max-plus algebra into a system of interval max-plus linear equations. The service cycle completion time of the networks is an eigenvalue of interval matrix in the system, also have been discussed in [5].

Recently, the fuzzy networks modelling has been developed. The activity times in a network is seldom precisely known, and then could be represented into the *fuzzy numbers* and then is called *fuzzy activity times*. In [6] have been discussed an algebra model of the acyclic fork-join queuing networks with fuzzy activity times using fuzzy number max-plus algebra into a system of fuzzy number max-plus linear equations. The eigenvalues of matrices over fuzzy number max-plus algebra, that is called fuzzy number max-plus eigenvalues have been discussed in [7].

Following the notion of the service cycle completion time of network analysis in [4] and [5] and using some results in the fuzzy number max-plus eigenvalues [7], this paper will discuss the application of fuzzy number max-plus eigenvalues on queuing networks with fuzzy activity times, especially on the service cycle completion time issue.

2. Fuzzy Number Max-Plus Eigenvalues

In this section we will review some concepts of fuzzy number max-plus eigenvalues. Further details can be found in [7]. We assume that readers have known some basic concepts of fuzzy set and fuzzy number [8], [9] and [10]. Further details can be found in [8], [9] and [10].

Definition 2.1 *Let* $\mathbf{F(R)}_{\tilde{\varepsilon}} := \mathbf{F(R)} \cup \{\tilde{\varepsilon}\}$, *where* $\mathbf{F(R)}$ *is set of all fuzzy numbers and* $\tilde{\varepsilon} := \{-\infty\}$, *with the* α*-cut of* $\tilde{\varepsilon}$ *is* $\varepsilon^{\alpha} = [-\infty, -\infty]$, $\forall\ \alpha \in [0, 1]$. *In* $(\mathbf{F(R)})_{\tilde{\varepsilon}}$, *we define the operations* $\tilde{\oplus}$ *and* $\tilde{\otimes}$ *as follow, for every* $\tilde{a}, \tilde{b} \in \mathbf{F(R)}_{\tilde{\varepsilon}}$ *and* $k \in \mathbf{R}^{+}$, *with* $a^{\alpha} = [\underline{a}^{\alpha}, \overline{a}^{\alpha}] \in \mathbf{I(R)}_{\max}$ *and* $b^{\alpha} = [\underline{b}^{\alpha}, \overline{b}^{\alpha}] \in \mathbf{I(R)}_{\max}$, *where* $\mathbf{I(R)}_{\max}$ *is interval max-plus algebra.*

i) $\tilde{a} \,\tilde{\oplus}\, \tilde{b} = \max(\tilde{a}, \tilde{b})$ *is fuzzy number with its* α*-cut is*
$(a \oplus b)^{\alpha} := [\underline{a}^{\alpha} \oplus \underline{b}^{\alpha}, \overline{a}^{\alpha} \oplus \overline{b}^{\alpha}]$, *for every* $\alpha \in (0, 1]$

ii) $\tilde{a} \,\tilde{\otimes}\, \tilde{b} = \tilde{a} \tilde{+} \tilde{b}$ *is fuzzy number with its* α*-cut is*
$(a \otimes b)^{\alpha} := [\underline{a}^{\alpha} \otimes \underline{b}^{\alpha}, \overline{a}^{\alpha} \otimes \overline{b}^{\alpha}]$, *for every* $\alpha \in (0, 1]$.

iii) $k \,\tilde{\otimes}\, \tilde{a}$ *is fuzzy number with its* α*-cut is*
$(k \otimes a)^{\alpha} := [k \otimes \underline{a}^{\alpha}, k \otimes \overline{a}^{\alpha}]$, *for every* $\alpha \in (0, 1]$.

We can show that α-cuts in this definition satisfied the conditions of α-cut of a fuzzy number. Since ($\mathbf{I}(\mathbf{R}_\varepsilon)$, $\overline{\oplus}$, $\overline{\otimes}$) is an idempotent comutative semiring, from the operations in $(\mathbf{F}(\mathbf{R})_{\tilde{\varepsilon}}$, we can show that ($\mathbf{F}(\mathbf{R})_\varepsilon$, $\tilde{\oplus}$, $\tilde{\otimes}$) is an idempotent comutative semiring, with neutral element is $\tilde{\varepsilon} = \{-\infty\}$ and unity element is $\tilde{e} = \{0\}$, with $e^\alpha = [0, 0]$, $\forall \alpha \in [0, 1]$). The idempotent comutative semiring $\mathbf{F}(\mathbf{R})_{\max} := (\mathbf{F}(\mathbf{R})_\varepsilon, \tilde{\oplus}, \tilde{\otimes})$ is called *fuzzy number max-plus algebra*, which is written as $\mathbf{F}(\mathbf{R})_{\max}$.

Definition 2.2 *Define* $\mathbf{F}(\mathbf{R})_{\max}^{m\times n} := \{ \tilde{A} = (\tilde{A}_{ij}) \mid \tilde{A}_{ij} \in \mathbf{F}(\mathbf{R})_{\max}$, *for* $i = 1, 2, ..., m$ *and* $j = 1, 2, ..., n$ }. *The elements of* $\mathbf{F}(\mathbf{R})_{\max}^{m\times n}$ *is called matrices over fuzzy number max-plus algebra*.

Further, this matrices would be called *fuzzy number matrices*. The operations $\tilde{\oplus}$ and $\tilde{\otimes}$ in $\mathbf{F}(\mathbf{R})_{\max}$ can be extended to the operations of fuzzy number matrices in $(\mathbf{F}(\mathbf{R})_{\max}^{m\times n}$. Specifically, for matrices $\tilde{A}$, $\tilde{B} \in \mathbf{F}(\mathbf{R})_{\max}^{n\times n}$ and $\tilde{\alpha} \in \mathbf{F}(\mathbf{R})_{\max}$ we define

$$(\tilde{\alpha} \tilde{\otimes} \tilde{A})_{ij} = \tilde{\alpha} \tilde{\otimes} \tilde{A}_{ij},\ (\tilde{A} \tilde{\oplus} \tilde{B})_{ij} = \tilde{A}_{ij} \tilde{\oplus} \tilde{B}_{ij} \text{ and } (\tilde{A} \tilde{\otimes} \tilde{B})_{ij} = \widetilde{\bigoplus_{k=1}^{n}} \tilde{A}_{ik} \otimes \tilde{B}_{kj}.$$

For every $\tilde{A} \in \mathbf{F}(\mathbf{R})_{\max}^{m\times n}$ and for some number $\alpha \in [0, 1]$ define *α-cut matrices of* $\tilde{A}$, that is interval matrices $A^\alpha = (A_{ij}^\alpha)$, with A_{ij}^α is the α-cut of $\tilde{A}_{ij}$ for every i and j. We notice that $A^\alpha \in \mathbf{I}(\mathbf{R})_{\max}^{m\times n}$, so according to the results of section 3, we have that $A^\alpha \approx [\underline{A}^\alpha, \overline{A}^\alpha]$. Moreover, for matrices $\tilde{A}$, $\tilde{B} \in (\mathbf{F}(\mathbf{R})_{\max}^{n\times n}$, where $A^\alpha \approx [\underline{A}^\alpha, \overline{A}^\alpha]$ and $B^\alpha \approx [\underline{B}^\alpha, \overline{B}^\alpha]$, we have that $\tilde{\alpha} \tilde{\otimes} \tilde{A}$, $\tilde{A} \tilde{\oplus} \tilde{B}$ and $\tilde{A} \tilde{\otimes} \tilde{B}$ is the fuzzy number matrices with their α-cut matrices are $(\alpha \otimes A)^\alpha \approx [\underline{\alpha} \otimes \underline{A}^\alpha, \overline{\alpha} \otimes \overline{A}^\alpha]$, $(A \oplus B)^\alpha \approx [\underline{A}^\alpha \oplus \underline{B}^\alpha, \overline{A}^\alpha \oplus \overline{B}^\alpha]$ and $(A \otimes B)^\alpha \approx [\underline{A}^\alpha \otimes \underline{B}^\alpha, \overline{A}^\alpha \otimes \overline{B}^\alpha]$, respectively.

Definition 2.3 *Let* $\tilde{A} \in \mathbf{F}(\mathbf{R})_{\max}^{n\times n}$. *The fuzzy number scalar* $\tilde{\lambda} \in \mathbf{F}(\mathbf{R})_{\max}$ *is called fuzzy number max-plus eigenvalues of matrices* $\tilde{A}$ *if there exist a fuzzy number vector* $\tilde{\mathbf{v}} \in \mathbf{F}(\mathbf{R})_{\max}^{n}$ *with* $\tilde{\mathbf{v}} \neq \tilde{\boldsymbol{\varepsilon}}_{n\times 1}$ *such that* $\tilde{A} \tilde{\otimes} \tilde{\mathbf{v}} = \tilde{\lambda} \tilde{\otimes} \tilde{\mathbf{v}}$. *The vector* $\tilde{\mathbf{v}}$ *is called fuzzy number max-plus eigenvectors matrces* $\tilde{A}$ *associaed with* $\tilde{\lambda}$.

Theorem 2.1 *Let* $\tilde{A} \in \mathbf{F}(\mathbf{R})_{\max}^{n\times n}$. *The fuzzy number scalar* $\tilde{\lambda}_{\max}(\tilde{A}) = \bigcup_{\alpha\in[0,1]} \tilde{\lambda}_{\max}^\alpha$, *where* $\tilde{\lambda}_{\max}^\alpha$ *is a fuzzy set in* $\mathbf{R}$ *with membership function* $\mu_{\tilde{\lambda}_{\max}^\alpha}(x) = \alpha\,\chi_{\tilde{\lambda}_{\max}^\alpha}(x)$, *where* $\chi_{\tilde{\lambda}_{\max}^\alpha}$ *is a characteristic function of the set* $[\lambda_{\max}(\underline{A}^\alpha), \lambda_{\max}(\overline{A}^\alpha)]$, *is a fuzzy number max-plus eigenvalues of matrices* $\tilde{A}$, *with* $\lambda_{\max}(\underline{A}^\alpha) = \bigoplus_{k=1}^{n} (\frac{1}{k} \bigoplus_{i=1}^{n} ((\underline{A}^\alpha)^{\otimes k})_{ii})$ *and* $\lambda_{\max}(\overline{A}^\alpha) = \bigoplus_{k=1}^{n} (\frac{1}{k} \bigoplus_{i=1}^{n} ((\overline{A}^\alpha)^{\otimes k})_{ii})$.

Proof: see [7].

3. Service Cycle Completion Time

We consider a network with n single-server nodes and customers of a single class. Further details can be found in [3] and [4]. The structure of the network is described by an oriented acyclic graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$, where the arcs determining the transition routes of customers. For every node i, we denote the sets of its immediate predecessors and successors respectively as $P(i) = \{ j \mid (j, i) \in \mathcal{A} \}$ and $S(i) = \{ j \mid (j, i) \in \mathcal{A} \}$.

Let $\tilde{a}_i(k)$ = fuzzy arrival time of kth customer at node i.

$\tilde{d}_i(k)$ = fuzzy departure time of kth customer at node i.

$\tilde{t}_{ik}$ = fuzzy service time of kth customer at server i.

We assumed that the network start operating at time zero, $\tilde{d}_i(0) = 0 = \tilde{0}$ and $\tilde{d}_i(k) = \varepsilon = \tilde{\varepsilon}$ for all $k < 0$, $i = 1, ..., n$. Let $\tilde{\mathbf{d}}(k) = [\tilde{d}_1(k), \tilde{d}_2(k), ..., \tilde{d}_n(k)]^T$, $\tilde{\mathbf{a}}(k) = [\tilde{a}_1(k), \tilde{a}_2(k), ..., \tilde{a}_n(k)]^T$. The explicit dynamic state equation of networks is given in theorem bellow.

Theorem 3.1 *Given the acyclic fork-join queuing networks with fuzzy number activity, with structure graph of the networks has the longest path p dan adjacency matrix* $\tilde{G}$. *The explicit dynamic state equation of networks is* $\tilde{\mathbf{d}}(k) = \tilde{A}(k) \tilde{\oplus} \tilde{\mathbf{d}}(k-1)$, where $\tilde{A}(k) = (\tilde{E} \tilde{\oplus} (\tilde{T}_k \tilde{\otimes} \tilde{G}))^p \tilde{\otimes} \tilde{T}_k$,

$$\tilde{E} = \begin{bmatrix} \tilde{0} & & \tilde{\varepsilon} \\ & \ddots & \\ \tilde{\varepsilon} & & \tilde{0} \end{bmatrix} \text{ and } \tilde{G}_{ij} = \begin{cases} 0 = \tilde{0}, & \text{if } j \in P(i) \\ \varepsilon = \tilde{\varepsilon}, & \text{otherwise} \end{cases}.$$

Proof: see [6]

Before we give an example, we remember about a special fuzzy number. A *triangular fuzzy number* $\tilde{a}$, which is written as TFN(a_1, a, a_2) or shortly (a_1, a, a_2), is a fuzzy number with membership function

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{x-a_1}{a-a_1}, & a_1 \le x \le a \\ \dfrac{a_2-x}{a_2-a}, & a \le x \le a_2 \\ 0, & others \end{cases}.$$

The *support of* $\tilde{a}$ is an open interval (a_1, a_2) and its α-cut is

$$a^{\alpha} = \begin{cases} [(a-a_1)\alpha + a_1, -(a_2-a)\alpha + a_2] & ,\alpha \in (0,1] \\ [a_1, a_2] & ,\alpha = 0 \end{cases}.$$

Example 3.1 *The acyclic fork-join queuing networks with n = 5 is shown in Figure 1 bellow.*

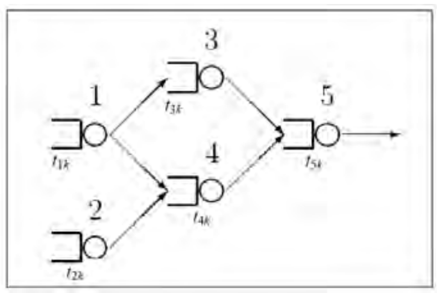


Figure 1. The acyclic fork-join queuing networks [6]

Let $\tilde{t}_{1k}$ = (2, 3, 4), $\tilde{t}_{2k}$ = (3, 4, 5), $\tilde{t}_{3k}$ = (5, 6, 6), $\tilde{t}_{3k}$ = (4, 4, 5), $\tilde{t}_{5k}$ = (3, 5, 6). We have

$$\begin{bmatrix} \tilde{d}_1(1) \\ \tilde{d}_2(1) \\ \tilde{d}_3(1) \\ \tilde{d}_4(1) \\ \tilde{d}_5(1) \end{bmatrix} = \begin{bmatrix} (2,3,4) & \tilde{\varepsilon} & \tilde{\varepsilon} & \tilde{\varepsilon} & \tilde{\varepsilon} \\ \tilde{\varepsilon} & (3,4,5) & \tilde{\varepsilon} & \tilde{\varepsilon} & \tilde{\varepsilon} \\ (7,9,10) & \tilde{\varepsilon} & (5,6,6) & \tilde{\varepsilon} & \tilde{\varepsilon} \\ (6,7,9) & (7,8,10) & \tilde{\varepsilon} & (4,4,5) & \tilde{\varepsilon} \\ (10,14,16) & (10,13,16) & (8,11,12) & (7,9,11) & (3,5,6) \end{bmatrix} \overline{\otimes} \begin{bmatrix} (0,0,0) \\ (0,0,0) \\ (0,0,0) \\ (0,0,0) \\ (0,0,0) \end{bmatrix} = \begin{bmatrix} (2,3,4) \\ (3,4,5) \\ (7,9,10) \\ (7,8,10) \\ (10,14,16) \end{bmatrix},$$

$$\begin{bmatrix} \tilde{d}_1(2) \\ \tilde{d}_2(2) \\ \tilde{d}_3(2) \\ \tilde{d}_4(2) \\ \tilde{d}_5(2) \end{bmatrix} = \begin{bmatrix} (2,3,4) & \tilde{\varepsilon} & \tilde{\varepsilon} & \tilde{\varepsilon} & \tilde{\varepsilon} \\ \tilde{\varepsilon} & (3,4,5) & \tilde{\varepsilon} & \tilde{\varepsilon} & \tilde{\varepsilon} \\ (7,9,10) & \tilde{\varepsilon} & (5,6,6) & \tilde{\varepsilon} & \tilde{\varepsilon} \\ (6,7,9) & (7,8,10) & \tilde{\varepsilon} & (4,4,5) & \tilde{\varepsilon} \\ (10,14,16) & (10,13,16) & (8,11,12) & (7,9,11) & (3,5,6) \end{bmatrix} \overline{\otimes} \begin{bmatrix} (2,3,4) \\ (3,4,5) \\ (7,9,10) \\ (7,8,10) \\ (10,14,16) \end{bmatrix} = \begin{bmatrix} (4,6,8) \\ (6,8,10) \\ (12,15,16) \\ (11,12,15) \\ (15,20,22) \end{bmatrix}.$$

$$\begin{bmatrix} \tilde{d}_1(3) \\ \tilde{d}_2(3) \\ \tilde{d}_3(3) \\ \tilde{d}_4(3) \\ \tilde{d}_5(3) \end{bmatrix} = \begin{bmatrix} (6,9,12) \\ (9,12,15) \\ (17,21,22) \\ (13,16,20) \\ (20,26,28) \end{bmatrix}, \begin{bmatrix} \tilde{d}_1(4) \\ \tilde{d}_2(4) \\ \tilde{d}_3(4) \\ \tilde{d}_4(4) \\ \tilde{d}_5(4) \end{bmatrix} = \begin{bmatrix} (8,12,16) \\ (12,16,20) \\ (22,27,28) \\ (16,20,25) \\ (25,32,34) \end{bmatrix}, \begin{bmatrix} \tilde{d}_1(5) \\ \tilde{d}_2(5) \\ \tilde{d}_3(5) \\ \tilde{d}_4(5) \\ \tilde{d}_5(5) \end{bmatrix} = \begin{bmatrix} (10,15,20) \\ (15,20,25) \\ (27,33,34) \\ (19,24,30) \\ (30,38,40) \end{bmatrix}, \ldots .$$

We consider the evolution of the system as a sequence of service cycles: the 1st cycle starts at the initial time, and it is terminated as soon as all the servers in the network complete their 1st service, the 2nd cycle is terminated as soon as the servers complete their 2nd service, and so on. Clearly, the fuzzy completion time of the *k*th cycle can be represented as

$$\max_i (\tilde{d}_i(k))$$

and the fuzzy service cycle completion time of the networks can be represented

$$\tilde{\gamma} = \lim_{k\to\infty} \frac{1}{k} \max_i (\tilde{d}_i(k)).$$

Theorema 3.2 *The acyclic fork-join queuing networks with fuzzy number activity with the explicit dynamic state equation* $\tilde{\boldsymbol{d}}$ (k) = $\tilde{A}$ (k) $\tilde{\oplus}$ $\tilde{\boldsymbol{d}}$ (k –1), *has the fuzzy service cycle completion time* $\tilde{\gamma} = \tilde{\lambda}_{\max}$ *(* $\tilde{A}$ *), that is an eigenvalue of* $\tilde{A}$.

Proof:

According to the Theorem 2 in [5] we have $\lim_{k\to\infty} \frac{1}{k} \max_i (d_i^{\alpha}(k))$ = [$\lambda_{\max}(\underline{A}^{\alpha})$, $\lambda_{\max}(\overline{A}^{\alpha})$] , $\forall \alpha \in$ [0,1], where $\lambda_{\max}(\underline{A}^{\alpha}) = \bigoplus_{k=1}^{n} (\frac{1}{k} \bigoplus_{i=1}^{n} ((\underline{A}^{\alpha})^{\otimes^k})_{ii})$ and $\lambda_{\max}(\overline{A}^{\alpha}) = \bigoplus_{k=1}^{n} (\frac{1}{k} \bigoplus_{i=1}^{n} ((\overline{A}^{\alpha})^{\otimes^k})_{ii}$. According to the [7] we know that [$\lambda_{\max}(\underline{A}^{\alpha})$, $\lambda_{\max}(\overline{A}^{\alpha})$] is a nested interval. Using the Decomposition Theorem in Fuzzy Sets, fuzzy number scalar $\tilde{\lambda}_{\max}(\tilde{A}) = \bigcup_{\alpha\in[0,1]} \tilde{\lambda}^{\alpha}_{\max}$, where $\tilde{\lambda}^{\alpha}_{\max}$ is a fuzzy set in **R** with membership function $\mu_{\tilde{\lambda}^{\alpha}_{\max}}(x) = \alpha\, \chi_{\tilde{\lambda}^{\alpha}_{\max}}(x)$, where $\chi_{\tilde{\lambda}^{\alpha}_{\max}}$ is a characteristic function of the set $[\lambda_{\max}(\underline{A}^{\alpha}), \lambda_{\max}(\overline{A}^{\alpha})]$, is a fuzzy number max-plus eigenvalues of matrices $\tilde{A}$. So we have, $\tilde{\gamma} = \lim_{k\to\infty} \frac{1}{k} \max_i (\tilde{d}_i(k)) = \tilde{\lambda}_{\max}(\tilde{A})$. ■

Example 3.2

From Example 3.1, using MATLAB computer program, we sketch the bounds α-cut of $\tilde{\lambda}_{\max}(\tilde{A})$ in the Figure 3.2 bellow.

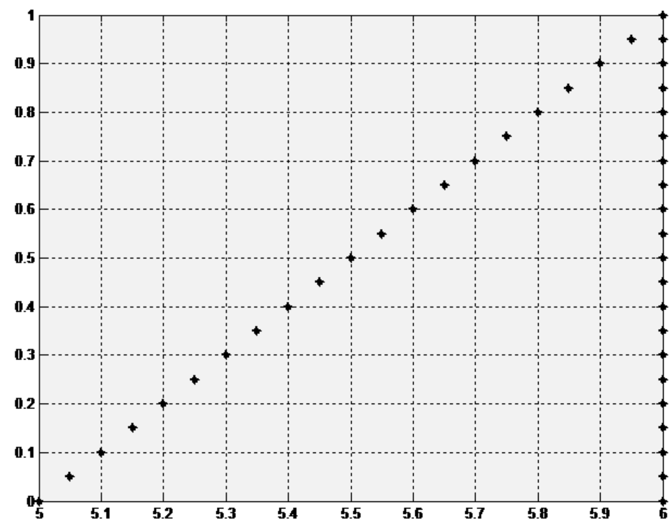


Figure 2. Sketch the bounds α-cut of $\tilde{\lambda}_{\max}(\tilde{A})$

From Figure 2, we can determine that $\tilde{\gamma} = \lim_{k\to\infty} \frac{1}{k} \max_i (\tilde{d}_i(k)) = \tilde{\lambda}_{\max}(\tilde{A})$ = TFN(5, 6, 6).

Conclusion

We can conclude that that the service cycle completion time of the acyclic fork-join queuing networks with fuzzy number activity times is an eigenvalue of matrices over fuzzy number max-plus algebra in the its explicit dynamic state equation.

References

[1] **F. Bacelli, *et al.*** *Synchronization and Linearity*, John Wiley & Sons, New York, 2001.
[2] **M. A. Rudhito.** *System Linear Max-Plus Waktu-Invariant*, Tesis: Program Pascasarjana Universitas Gadjah Mada, Yogyakarta, 2003
[3] **N.K. Krivulin.** *Algebraic Modelling and Performance Evaluation of Acyclic Fork-Join Queueing Networks.* Advances in Stochastic Simulation Methods, Statistics for Industry and Technology, (2000), 63-81.
[4] **M. A. Rudhito and A. Suparwanto.** *Pemodelan Aljabar Max-Plus dan Evaluasi Kinerja Jaringan Antrian Fork-Join Taksiklik dengan Kapasitas Penyangga Takhingga*. Prosiding Seminar Nasional Sains dan Pendidikan Sains UKSW Salatiga (2008), B3-1 – B3-13.
[5] **M. A. Rudhito, *et.al.*** *Penerapan Aljabar Max - Plus Interval pada Jaringan Antrian dengan Waktu Aktifitas Interval.* Prosiding Seminar Nasional Aljabar, Pengajaran dan Terapannya UNY Yogyakarta (2009), M2 11-18.
[6] **M. A. Rudhito, *et.al.*** *Penerapan Aljabar Max-Plus Bilangan Kabur pada Jaringan Antrian dengan Waktu Aktifitas Kabur*, Prosiding Seminar Nasional Penelitian, Pendidikan, Dan Penerapan MIPA UNY Yogyakarta (2009), M-265 – M-270.
[7] **M. A. Rudhito, *et.al.*** *Eigenvalues And Eigenvectors Of Matrices Over Fuzzy Number Max-Plus Algebra*, Proceeding the 3rd International Conference on Mathematics and Statistics IPB Bogor (2008), 195 – 202.
[8] **H.J. Zimmermann,** *Fuzzy Set Theory and Its Applications,* Kluwer Academic Publishers, Boston, 1991.
[9] **K.H. Lee,** *First Course on Fuzzy Theory and Applications,* Spinger-Verlag, Berlin Heidelberg, 2005.
[10] **F. Susilo,** Himpunan dan Logika Fuzzy serta Aplikasinya edisi kedua, Graha Ilmu, Yogyakarta, 2006.